Capacity analysis of an ARQ scheme for multimedia DS-CDMA systems

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Abstract: A multimedia DS-CDMA system using an ARQ reverse link for data transmission is proposed and analysed. The system capacity is evaluated for several error-detecting codes (EDCs) such as the Hamming codes, the Golay code, the extended Golay code and two kinds of BCH codes. A simple new model based on the asymptotic gain for block and convolutional codes is proposed. Using this asymptotic gain model, analytical expressions for the system capacity are derived, which makes the system capacity evaluation easier and faster. The proposed system allows a greater number of data users by using more powerful block EDCs, instead of the Hamming class. The convolutional code efficiency in this system is also discussed.

1 Introduction

To increase the quality and assist the demand for multimedia wireless communication services, Lee and Kim [1] proposed the use of Hamming error-detecting codes in an ARQ scheme to protect data packets on the data channel of an IS-95 type DS-CDMA system.

In this system, a 1/2 rate convolutional error-correcting code is used in the voice channel to increase the system capacity. The transmission quality is not perceptibly affected if data packets undergo short delays. Therefore an ARQ scheme is a very suitable enhancement for the data channel.

Generally, an ARQ scheme makes use of an error-detection code (EDC) and a feedback channel (assumed error-free). Information is encoded and sent on the channel. If an error is detected, a NACK (negative acknowledgement) is transmitted on the feedback channel and, after the NACK is received, the transmitter resends the same message. This operation will happen whenever the receiver transmits NACKs.

In this paper, we analyse the capacity of this system using the Golay code (23, 12, 7), the extended Golay code (24, 12, 8) and BCH codes with $d_{min} = 5$ and $d_{min} = 7$ (EDC). The system capacity is evaluated by using a simple model based on the asymptotic gain of block EDCs. This model simplifies the analysis and calculation, and it presents excellent agreement with the BER (bit error rate) approach used in [1].

2 Asymptotic gain of block codes

A binary block code (n, k, d_{min}) which transforms k information bits in n encoded bits has a code rate $R_B = k/n$. d_{min} is the minimum Hamming distance between any pair of codewords. Therefore error detection is always possible if the number of bit errors in a word does not exceed $d_{min} - 1$.

IEE Proceedings online no. 2000574

DOI: 10.1049/ip-com:20000574

Paper first received 13th August 1999 and in revised form 11th May 2000

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For an ARQ scheme, the probability of undetected bit error is approximately given by [2]

$$P_{ube} \approx \sum_{j=d_{min}}^{n} \frac{A_j}{k} p^j (1-p)^{n-j} \quad (1)$$

where A_j is the number of codewords of weight j and p is the bit-error probability of the coded system (with no ARQ). Assuming DPSK modulation, p is given by

$$p = \frac{1}{2} \exp(-R_B \frac{E_b}{N_0}) \quad (2)$$

where E_b/N_0 is the signal-to-noise ratio (SNR) of the uncoded system. Substituting eqn. 2 in eqn. 1 and assuming that $p << 1$, we can approximate eqn. 1 as

$$P_{ube} \approx \frac{A_{d_{min}}}{k} p^{d_{min}} = \frac{A_{d_{min}}}{k} \frac{1}{2^{d_{min}}} \exp(-R_B d_{min} \frac{E_b}{N_0}) \quad (3)$$

The uncoded bit-error probability is expressed as

$$P_b(E) = \frac{1}{2} \exp(-\frac{E_b}{N_0}) \quad (4)$$

Observing the exponents of eqns. 2–4, we conclude that the gain in SNR, obtained by using an ARQ scheme with a block code compared to the uncoded system, is $R_B d_{min}$ and compared to the coded system is d_{min}. The asymptotic coding gain, in terms of SNR, is valid only for high values of E_b/N_0.

3 CDMA model

The capacity of a DS-CDMA cellular system is limited by the interference caused by the users at the home cell and by the users of other cells. We consider a system that supports voice and data services. The signal to noise ratio of the received data and voice signals (see Figs. 1 and 2) may be expressed, respectively, as [1]

$$SNR_d = (\frac{E_b}{N_0})_d = \frac{G_d S_d}{\sum_{i=1}^{N_v} \alpha S_{v,i} + \sum_{j=1}^{N_d - 1} S_{d,j} + I + N} \quad (5)$$

$$SNR_v = (\frac{E_b}{N_0})_v = \frac{G_v S_v}{\sum_{i=1}^{N_v-1} \alpha S_{v,i} + \sum_{j=1}^{N_d} S_{d,j} + I + N} \quad (6)$$

where G_d and G_v are, respectively, the data and the voice user processing gains, S_d and S_v are, respectively, the received power of the desired data and voice users, $S_{v,i}$ is the received power of the ith voice user, $S_{d,j}$ is the received power of the jth data user, N_v and N_d are, respectively, the number of voice and data users, I is the other cell interference, N is the background noise and α is the voice-activity factor.

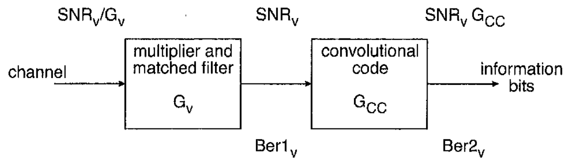

Fig. 1 *Receiver for voice users*

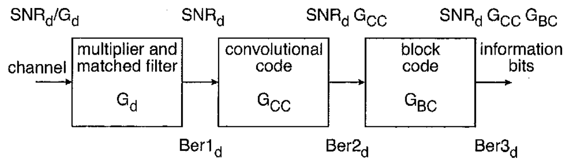

Fig. 2 *Receiver for data users*

Assuming perfect power control and 'f' as the fraction of interference caused by the other cells, we may write [1]:

$$S_v = AS_d \quad (7)$$

where

$$A = \frac{G_d \left(\frac{E_b}{N_0}\right)_d^{-1} + 1 + f}{G_v \left(\frac{E_b}{N_0}\right)_v^{-1} + (1+f)\alpha} \quad (8)$$

Considering the interference to be prevalent over the noise, from eqns. 5–7 we have

$$\alpha N_v + A^{-1} N_d \le \left(\frac{G_d}{(\frac{E_b}{N_0})_d (1+f)} + 1 \right) A^{-1} \quad (9)$$

Eqn. 9 expresses the maximum number of voice and data users per cell, i.e. the capacity of the system.

4 BER approach

Fig. 1 illustrates the voice receiver in an IS-95 type DS-CDMA system. Information bits are encoded by a convolutional code and then spread and transmitted. At the receiver, the inverse process is executed.

The data receiver is shown in Fig. 2. The data signal is first encoded by a block EDC and then encoded by a convolutional code identical to that used in Fig. 1. The sequence is then spread and transmitted. We denoted the bit error rate (BER) before the convolutional code as $Ber1_d$, the BER before the block EDC as $Ber2_d$ and the allowable information BER as $Ber3_d$. Assuming hard-decision decoding of the convolutional code, these BERs are related by [2, 3]:

$$Ber3_d(Ber2_d) \cong \sum_{j=d_{\min}}^{n} \frac{A_j}{k} (Ber2_d)^j (1 - Ber2_d)^{n-j} \quad (10)$$

$$Ber2_d(Ber1_d) \cong \frac{1}{k_o} \sum_{d=d_{\text{free}}}^{\infty} B_d \left[2\sqrt{Ber1_d(1 - Ber1_d)} \right]^d \quad (11)$$

where (n_o, k_o) are the convolutional code parameters, d_{free} is the minimum free distance of the convolutional code, B_d is the total number of non-zero information bits on all paths with weight d in the trellis diagram of the convolutional code.

Substituting $Ber2_d$ for $Ber2_v$ (allowable voice BER) and $Ber1_d$ for $Ber1_v$ (BER before the convolutional encoding), eqn. 11 can also be used for the voice channel. Then we can relate $Ber1_d$ with SNR_d and $Ber1_v$ with SNR_v, which allows us to evaluate the system capacity using eqn. 9.

5 Asymptotic approach

Figs. 1 and 2 also illustrate the SNRs after every reception stage. G_v and G_d are, respectively, the DS-CDMA SNR processing gains for voice and data signals, G_{CC} is the SNR gain due to the convolutional code and G_{BC} is the SNR gain obtained by the block encoding stage. For the data channel, it is assumed that $Ber3_d = 10^{-9}$, so high values of SNR are expected after the block EDC. Therefore the gain in SNR of the block EDC is given approximately by its asymptotic gain in terms of SNR compared to the coded system (see Section 2). Therefore

$$G_{BC} \approx d_{\min} \quad (12)$$

On the other hand, low values of SNR are expected for the convolutional code output SNR. So, instead of the asymptotic gain, a polynomial interpolation is used to express G_{CC} as a function of its output SNR, here denoted by x. For instance, the SNR gain G_{CC} of the IS-95 voice reverse link convolutional code can be well fitted by the polynomial

$$G_{CC}(x) \approx 0.08595 + 0.42796x - 0.01227x^2 \quad (13)$$

for $2 < x < 14$. This code has constraint length $K = 9$, code rate $R_C = k_o/n_o = 1/2$, $d_{\text{free}} = 12$ and generator polynomial, in octal, $g = ([753\ 561])$.

Then, for a given $Ber3_d$ and a DPSK modulation, we can approximately express 'x', SNR_d and SNR_v as:

$$x = \frac{-\ln(2Ber3_d)}{d_{\min}} \quad (14)$$

$$SNR_d = \frac{x}{G_{CC}(x)} \quad (15)$$

$$SNR_v = \frac{-\ln(2Ber2_v)}{G_{CC}(-\ln(2Ber2_v))} \quad (16)$$

We can rewrite the capacity of the system by using eqns. 15 and 16 as

$$\alpha N_v + A^{-1} N_d \le \left(\frac{G_d}{\frac{-\ln(2Ber3_d)}{d_{\min} G_{CC}(\frac{-\ln(2Ber3_d)}{d_{\min}})} (l + f)} + 1 \right) A^{-1} \quad (17)$$

where A is given by:

$$A = \frac{G_d \left(\frac{\frac{-\ln(2Ber3_d)}{d_{\min}}}{G_{CC}(\frac{-\ln(2Ber3_d)}{d_{\min}})} \right)^{-1} + 1 + f}{G_v \left(\frac{-\ln(2Ber2_v)}{G_{CC}(-\ln(2Ber2_v))} \right)^{-1} + (1+f)\alpha} \quad (18)$$

This asymptotic approach makes the system capacity evaluation easier and faster than the BER approach.

6 Results

Parameters similar to those of IS-95 type DS-CDMA [1] are used, i.e. $\alpha = 0.453$, $Ber3_d = 10^{-9}$, $Ber2_v = 10^{-3}$ and $f = 0.55$. A negligible background noise is assumed. Information data and voice rates are 9600bits/s and the spreading bandwidth is 1.2288MHz. We also assume a small probability of retransmission, which is a reasonable assumption once the BERs are small.

The data receiver can also be analysed considering the total SNR reception gain Γ_D, see Fig. 2:

$$\Gamma_D = G_{CC}G_{BC}G_d \approx G_{CC}d_{\min}R_BR_CG_0 \qquad (19)$$

where $G_D = G_0R_BR_C$ and G_0 is the processing gain for data users without codes. Eqn. 19 indicates that, for a given convolutional code, it is preferable to use EDCs that maximise the product $R_Bd_{\min}$. We may also conclude that, for a given EDC, the convolutional code works efficiently when

$$R_CG_{CC} > 1 \Rightarrow G_{CC} > \frac{1}{R_C} \qquad (20)$$

For the convolutional code used, for instance when $R_C = 1/2$, G_{CC} must be greater than 2, otherwise the presence of the convolutional code in the receiver will degrade the overall performance. This phenomenon is observed mainly when high $d_{\min}$ EDCs are used, as some of the following results suggest (Fig. 3 and Table 1). Therefore data reception without the convolutional code can be, in some cases, a better alternative for system implementation.

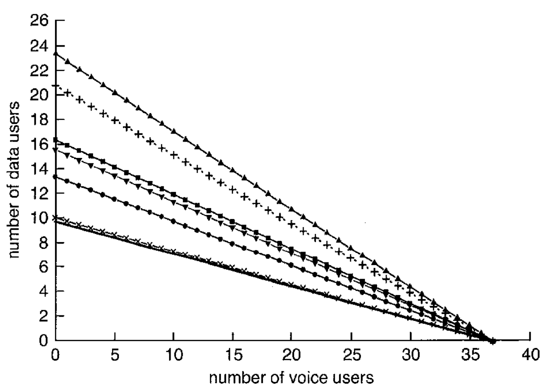

Fig. 3 *Capacity bound with a BCH (d_{min} = 7) block code obtained via the BER approach*
Without convolutional code:
—▲— (127, 106)
- -+- - (63, 45)
—■— (31, 16)
With convolutional code:
—▼— (127, 106)
—●— (63, 45)
—×— (31, 16)
——— no ARQ

Fig. 3 shows the system capacity, obtained via the BER approach, for some BCH codes with $d_{\min} = 7$ and when no ARQ scheme is used. We can observe the capacity gains provided by the ARQ scheme, especially when the data receiver has no convolutional code.

In Fig. 4, the capacity bounds of the system are showed for some BCH codes with $d_{\min} = 5$. The accuracy of the asymptotic approach when compared to the BER approach is also shown in that Figure.

As illustrated in Figs. 3 and 4, the maximum number of data users N_d(max) can be used as an indication of the maximum capacity of the system. Table 1 shows N_d(max), obtained via the BER and asymptotic approaches for several block EDCs, such as some Hamming codes, the Golay (23, 12, 7) code, the extended Golay (24, 12, 8) code and some BCH codes with $d_{\min} = 5$ and $d_{\min} = 7$. Table 1 also presents, for the referred EDCs, the maximum number of data users when the receiver does not employ the convolutional code.

Table 1: Maximum number of data users for several block codes

	Maximum number of data users $N_d(max)$			
	With convolutional codes		Without convolutional codes	
	BER approach	Asymptotic approach	BER approach	Asymptotic approach
Hamming				
(7, 4, 3)	9.47	9.46	7.97	8.06
(15, 11, 3)	11.78	11.86	9.69	10.07
(31, 26, 3)	13.23	13.42	10.65	11.37
(63, 57, 3)	14.075	14.4	11.11	12.19
(127, 120, 3)	14.528	15	11.25	12.68
(255, 247, 3)	14.7389	15.35	11.21	12.98
(511, 502, 3)	14.803	15.55	11.07	13.15
(1023, 1013, 3)	14.784	15.67	10.89	13.24
(2047, 2036, 3)	14.719	15.74	10.68	13.302
Golay				
(23, 12, 7)	10.03	10.11	15.71	16.05
Extended Golay				
(24, 12, 8)	9.76	9.9	16.881	17.49
BCH (d_{min} = 5)				
(15, 7, 5)	8.76	8.71	10.76	10.62
(31, 21, 5)	12.16	12.19	14.39	14.96
(63, 51, 5)	14.199	14.37	16.052	17.68
(127, 113, 5)	15.374	15.7	16.722	19.34
(255, 239, 5)	16.0088	16.48	16.8192	20.32
(511, 493, 5)	16.3421	16.94	16.7262	20.88
BCH (d_{min} = 7)				
(31, 16, 7)	9.989	10.01	16.353	15.89
(63, 45, 7)	13.336	13.47	20.7325	21.61
(127, 106, 7)	15.358	15.57	23.308	25.08

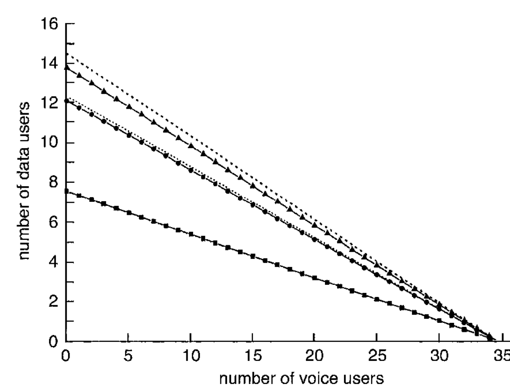

Fig. 4 *Capacity bound with a BCH (d_{min} = 5) block code*
—■— (15, 7) BER approach
·········· (15, 7) asymptotic approach
—●— (63, 51) BER approach
·········· (63, 51) asymptotic approach
—▲— (511, 493) BER approach
- - - - (511, 493) asymptotic approach

Differently from [1], we have considered for the convolutional code the 16 first terms of B_d [4] in eqn. 11. This fact justifies the slightly different Hamming codes N_d(max) values, when compared to the ones obtained in [1].

We can observe that BCH codes with d_{min} = 7, without the convolutional code, present the best performance among the codes presented in this paper. We can also realise that the values obtained by the asymptotic approach present good agreement with the ones obtained by the BER approach. Another conclusion is that, for high d_{min} codes, data reception without the convolutional code increases the system capacity for the specified $Ber3_d$.

7 Conclusions

We can claim that, for the proposed system, an ARQ scheme on the data channel can provide significant gain in the capacity of CDMA systems. A greater number of data users is achieved by using more powerful block EDCs, instead of the Hamming class analysed in [1], e.g. the BCH codes with d_{min} =7. A new model based on the asymptotic coding gain for block EDCs was proposed. Using this asymptotic gain model, analytical expressions for the system capacity were derived, which made the system capacity evaluation easier and faster than the BER approach evaluation used in [1]. We may also conclude that, for a system with $Ber3_d = 10^{-9}$ and powerful block codes, the use of the convolutional code at the data receiver is shown to be very inefficient.

8 Acknowledgment

This work was in part supported by Fundação de Amparo à Pesquisa do Estado de São Paulo – FAPESP. The authors wish to thank Dr. R. Baldini Filho for helpful suggestions.

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